Gain Enhancement in Cascaded Fiber Parametric Amplifier with Quasi-Phase Matching: Theory and Experiment

Jaeyoun Kim, Özdal Boyraz, Student Member, IEEE, Jin H. Lim, and Mohammed N. Islam

Abstract—We report a novel gain enhancement scheme for fiber-optic parametric amplifiers utilizing cascaded amplification and quasi-phase matching (QPM). The theory and method of QPM for four-wave mixing (FWM) are developed for the first time to our knowledge. In experimental implementations of the theory, we achieve >12 dB gain improvement in a three-stage dispersion-shifted (DS) fiber parametric amplifier. A 16-dB overall gain is obtained with 11 nm separation between zero-dispersion wavelength and pump wavelength. The experimental results show good agreement with theory and simulations. The influence of QPM on spectral characteristics of parametric gain is investigated with numerical simulations.

Index Terms—Nonlinear optics, nonlinear wave propagation, optical fiber amplifiers, optical fiber communication, optical fibers, optical frequency conversion, optical parametric amplifiers, optical propagation in nonlinear media.

I. INTRODUCTION

PTICAL amplifiers operating outside of erbium-doped fiber amplifier (EDFA) gain bandwidth are important for future broadband WDM systems. The parametric amplification (PA) process has been under intense research because it can amplify optical signals at arbitrary wavelengths [1], [2]. In addition, the generation of phase-conjugate wave during PA process is a promising method for wavelength conversion that will play an important role in multi-wavelength optical networks. In practice, the low PA gain resulting from the weak nonlinearity of optical fiber and the requirement for phase matching have hindered its applications. Pumping near zero-dispersion wavelength (λ_o) [2] or the use of high nonlinearity fiber [3], [4] have been proposed to enhance the gain. However, the unavoidable fluctuation of λ_o turns out to be detrimental to near- λ_o pumping scheme [5], [6]. To avoid the resultant gain fluctuation, a large separation (>10 nm) between λ_o and pump wavelength (λ_p) is preferred at the expense of higher dispersion and lowered gain. High nonlinearity fiber is not readily available for a variety of wavelengths and often causes extra loss and handling problems.

In this paper, we enhance the low PA gain by cascading the PA processes. Simply extending the length of PA gain fiber results in phase mismatch and periodic fluctuation of the gain. To overcome the problem, we utilize the quasi-phase matching (QPM)

The authors are with Optical Sciences Laboratory, Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI 48109-2122 USA.

Publisher Item Identifier S 0733-8724(01)00744-7.

concept that is popular in second-harmonic generation. For the first time to the best knowledge of the authors, we have developed the QPM theory for four-wave mixing (FWM) case. The theory gives information on the proper phase relation for maximum gain, the changes in phase relation during PA process, and its compensation. Similar theories have been reported in [4] and [7] for limited applications. We generalize them by obtaining explicit QPM conditions for single wavelength (called "QPM target wavelength") that can be applied to all wavelengths. The validity of the theory is verified experimentally and the effects of QPM on a broad range of wavelengths are investigated with simulations.

This paper is organized as follows. In Section II, we develop the theory and method for QPM in FWM. In Section III, the implementation and experimental results of QPM PA are presented and compared with theoretical predictions. Discussions on the spectrum and other properties of QPM PA follow in Section IV.

II. THEORY

The QPM by periodic inversion of domain has long been utilized in $\chi^{(2)}$ media to enhance the conversion efficiency. In inversion-symmetric $\chi^{(3)}$ media, including optical fiber, QPM requires change of a parameter other than domain polarity. The theory and experiment of QPM by periodic changes in optical power are reported in [8] and [9] emphasizing the generation of multiple-order sidebands rather than gain enhancement. The periodic change of fiber dispersion is utilized in FWM process for sideband suppression in fiber links [7] and compensation of dispersion in PA [4], [6]. However, the detailed theory and method of QPM for cascading PA processes have not been reported yet.

The basic concept of cascaded PA with QPM is summarized in Fig. 1. The dotted curve shows the parametric gain of a non-QPM PA as a function of fiber length. Similar to second-harmonic generation process in $\chi^{(2)}$ medium, the signal gain oscillates depending on the phase relation among the waves. To enhance the parametric gain, it is necessary to stop PA processes at each maximum gain point and introduce additional phase shift $\Delta \varphi_{add}$ to restore the phase relation as described in Fig. 2. The solid curve in Fig. 1 shows the enhanced gain of cascaded PA when adequate phase adjustment is introduced at each maximum gain point.

Implementation of QPM PA requires the knowledge of two parameters: 1) the amount of relative phase shift $\Delta \varphi_{add}$ and 2) the point of its introduction. In this work, we theoretically

Manuscript received March 7, 2000; revised October 11, 2000. This work was supported by the HRLD Foundation.

where

$$a = \cosh(g \cdot z) + i \cdot \frac{\pi}{2g} \cdot \sinh(g \cdot z)$$

$$b = i \cdot \frac{\gamma P_p}{g} \cdot e^{i \cdot 2\Phi_{po}} \cdot \sinh(g \cdot z)$$

$$\kappa = \Delta k + 2 \cdot \gamma P_p$$

$$g = \sqrt{(\gamma P_p)^2 - \left(\frac{\kappa}{2}\right)^2}$$
(3)

ĸ

and Φ_p is the nonlinear phase of the pump wave accumulated in the PA process and $|A_{so,co}|$ and $\varphi_{so,co}$ are the initial magnitude and phase of the signal and the conjugate wave, respectively [4]. The matrix elements a and b are complex numbers that determine the changes in amplitude and phase of signal and conjugate waves. They can also be represented by their magnitudes |a|, |b| and phases φ_a, φ_b , respectively. g is the parametric gain of the PA. In periodic gain regime ($\lambda_p < \lambda_o$), Δk is always positive and consequently g is always purely imaginary. Therefore the hyperbolic functions in (3) can be converted to corresponding trigonometric functions.

The maximum gain point for introduction of additional phase shift can be calculated by obtaining $|A_s(z)|^2$ in (2)

$$|A_{s}(z)|^{2} = |a|^{2} \cdot |A_{so}|^{2} + |b|^{2} \cdot |A_{co}|^{2} + 2 \cdot |a||b||A_{so}||A_{co}|$$

$$\cdot \cos(\varphi_{a} + \varphi_{so} - \varphi_{b} + \varphi_{co}).$$
(4)

The first two terms are maximized at the point where $|a|^2 = 1 + [(\kappa/2|g|)^2 - 1]\sin^2|g|z$ and $|b|^2 = (\gamma^2 P_p^2/|g|^2)\sin^2|g|z$ reach the maximum. They can be maximized simultaneously at L_{max} defined as

$$L_{\max} = \pi/2|g| \tag{5}$$

provided that $\kappa^2 > 4|g|^2$. Knowing that $|g|^2 = (\Delta k/2 + \gamma P)^2 - \gamma P^2$ when $\lambda_p < \lambda_o$ and using the definition of κ , it is straightforward to show that $\kappa^2 > 4|g|^2$ regardless of the magnitude of κ . L_{max} is analogous to the coherence length of second-harmonic generation. Due to the symmetric nature of the solution, the conjugate is maximized at the same length.

To determine the amount of additional phase shift, we first need to know both the initial phase relation that maximizes the signal power and then the change in phase relation caused by propagation and PA process. $\Delta \varphi_{add}$ is determined from the difference between them. The former can be obtained directly from the third term of (4). The expression is complicated in general. It can be simplified, however, by setting $z = L_{max}$. The resultant initial phase relation that maximizes the PA gain at L_{max} is

$$\Delta \varphi_o \equiv \varphi_{so} + \varphi_{co} - 2 \cdot \Phi_{po} = \pm 2n \cdot \pi \tag{6}$$

where n is an integer. The induced change in phase relation is obtained by calculating $\varphi_s + \varphi_c - 2 \cdot \Phi_p$ at $z = L_{\text{max}}$ using (2) and (3). φ_s , φ_c , and Φ_p are phase angles of signal, conjugate, and pump wave, respectively. The result depends on the initial phases of signal and conjugate. Under the assumption that $\Delta\varphi_o = \pm 2n \cdot \pi$ initially, it is easy to show that $\varphi_s + \varphi_c - 2 \cdot \Phi_p$ becomes $-\pi$ at L_{max} . In practice, $\Delta\varphi_o = \pm 2n \cdot \pi$ can be accomplished by fine-tuning of polarization controllers at the input of the PA.



Fig. 2. Concept of phase adjustment for QPM in FWM case.

find them under the assumptions of a FWM process within periodic gain regime ($\lambda_p < \lambda_o$) in which three wavelength components are involved without pump depletion. Nonlinear effects caused by signal and conjugate waves are also ignored. The coupled-wave equations that govern the interactions are [1]

$$\frac{dA_{s,c}}{dz} = i \cdot \left[(k_{s,c} + 2 \cdot \gamma P_p) \cdot A_{s,c} + \gamma P_p e^{i \cdot 2 \cdot \Phi_{po}} \cdot A_{c,s}^* \right] (1a)$$

$$\frac{dA_p}{dz} = i \cdot \gamma P_p \cdot A_p$$
(1b)

where

- $K_{s,c}$ propagation constants of signal and conjugate;
- γ nonlinear factor of the fiber;
- P_p peak pump power;
- ϕ_{po} initial phase of the pump wave.

 A_s , A_c , and A_p stand for signal, conjugate, and pump wave, respectively. Equation (1) has exact solution that can be best expressed in matrix form

$$\begin{bmatrix} A_s(z) \\ A_c^*(z) \end{bmatrix} = \begin{bmatrix} e^{i \cdot \Phi_p} & 0 \\ 0 & e^{-i \cdot \Phi_p} \end{bmatrix} \cdot \begin{bmatrix} a & b \\ b^* & a^* \end{bmatrix} \cdot \begin{bmatrix} |A_{so}| \cdot e^{+i \cdot \varphi_{so}} \\ |A_{co}| \cdot e^{-i \cdot \varphi_{co}} \end{bmatrix}$$
(2)







Fig. 3. Experimental setup (PC: Polarization Controller, LD: Laser Diode, EOM: Electro-Optic Modulator, OBPF: Optical Bandpass Filter, Det: Detector, OSC: Oscilloscope).

We can conclude that the QPM in FWM is achieved by ensuring that input waves are in proper phase relation and subsequently introducing additional relative phase difference of

$$\Delta \varphi_{\text{add}} \equiv \Delta \varphi_s + \Delta \varphi_c - 2 \cdot \Delta \Phi_p = \pm \pi \tag{7}$$

at each $L_{\text{max}} = \pi/2|g|$. $\Delta\varphi_s$, $\Delta\varphi_c$, and $\Delta\Phi_p$ are added phase shifts in signal, conjugate, and pump, respectively. $\Delta\varphi_{\text{add}}$ can be achieved by splicing a short section of high-dispersion fiber or a proper choice of phase response in all-pass optical filters. Note that the gain enhancement occurs only at QPM target wavelength that satisfies the condition of (5). The influences of QPM on other wavelength components will be discussed in Section IV. It is also important to note that the gain maximization conditions are distinct from previous experimental [10] and numerical [11] results due to differences in problem definition. Perfect phase matching is assumed in [10] and pump depletion is considered in [11].

III. EXPERIMENT

We experimentally verify the enhancement of the gain in QPM PA. Fig. 3 shows the experimental setup. For PA gain fiber, sections of dispersion-shifted (DS) fiber with $\lambda_o = 1561$ nm and dispersion-slope of 0.07 ps/nm²-km are used. For both fibers, the value of nonlinear factor γ is 2.2 W⁻¹km⁻¹. We set the $\lambda_o - \lambda_p$ separation to 11 nm to avoid the detrimental effect of λ_o fluctuation. Two laser diodes with wavelength of 1550 nm and 1556 nm are used as pump and signal, respectively. To maximize the peak pump power at the EDFA output, the pump wave is modulated to have 50 ns width with 50:1 duty-cycle by an electro-optic modulator. The two waves are combined by a 70:30 coupler and amplified by a high-power EDFA with 1 W saturation power. At the input to the PA gain fiber, the quasi-CW pump has 16 W peak power. The fact that the signal power is only 0.7% of the pump power justifies the assumptions made in the previous section.

Given the QPM target wavelength at 1556 nm, we set the length of PA gain fiber section to 50 m that is the maximum gain point according to (5). To introduce $\Delta \varphi_{add}$, we choose to use a section of SMF-28 fiber for its simplicity and availability. In highly dispersive SMF-28 fiber, only the linear phase shift is considered while the effect of nonlinear phase shift is ignored. The length of SMF-28 to introduce $\Delta \varphi_{add} = \pm \pi$ is calculated



Fig. 4. Gain enhancement versus SMF-28 fiber length. The gain enhancement is maximized at a length that makes $\Delta \varphi_{add} = -\pi$. 0.4 dB splicing loss is assumed in simulations.

from the relation $\Delta \varphi_{add} \equiv \Delta k \cdot L_{SMF-28}$ and the standard linear phase mismatch formula

$$\Delta k = k_s + k_c - 2 \cdot k_p = -2\pi c \cdot D_p \cdot (\lambda_p / \lambda_s - 1)^2$$

where D_p is the dispersion of the fiber at the pump wavelength. Substitution of experimental parameters shows that $\Delta k \cong -0.475 \text{ [m}^{-1}\text{]}$ and $\Delta \varphi_{\text{add}} = -\pi$ is obtained with 6.6 m of SMF-28 fiber. With the basic information obtained so far, we implement a two-stage (one SMF-28 section) and a three-stage (two SMF-28 sections) QPM PA. At the output of PA, the signal portion is selected by an optical bandpass filter and detected with a photodetector. Since PA occurs only when the pump is present, the gain is calculated as a ratio of the peak and the trough. The contributions of ASE and residual pump are also measured and excluded from gain calculations. Along with the experiments, we have also simulated the PA processes using the matrix solution (2). The propagation loss of the fiber is not considered because of the shortness of the fiber in the setup (<200 m). However, the splicing losses between two different types of fibers are included in the simulations as a power loss. Due to the technical difficulty in measuring the splicing loss, it is the only assumed variable in our simulations. For most cases, we use 0.4 dB power loss.

Fig. 4 shows the gain enhancement of the two-stage QPM PA as a function of SMF-28 fiber length that is varied from 0 to 18 m. The corresponding values of $\Delta \varphi_{add}$ are also plotted for reference. The maximum gain enhancement of 6.1 dB is achieved with 6 m of SMF-28 that sets $\Delta \varphi_{add}$ to $-\pi$ radian approximately. The gain enhancement is a periodic function of SMF-28 length since the integer multiple of 2π in $\Delta \varphi_{add}$ does not contribute to QPM. The mismatch between simulation and experimental data can be explained by high splicing loss between DS and SMF-28 fiber. Fig. 5 shows the measured gain along the fiber. Two circles near the dotted-curves show the effect of QPM when 6 m or 10 m sections of SMF-28 is inserted at the 50 m point. While the insertion of 6 m SMF-28 enhances



Fig. 5. Experimental and simulation results of a two-stage PA (Solid: PA gain without QPM, Dotted: PA gain with QPM).

gain by 6.1 dB, the 10 m SMF-28 results in much less enhancement due to improper phase adjustment. Simulation and experimental data of non-QPM PA using the same setup is included for comparison.

The experimental results of three-stage QPM PA are shown in Fig. 6 along with the simulation curve. Considering the higher loss from multiple splicing, we increase P_p to 17 W and accordingly reduce the length of the second stage DS fiber to 40 m. The measured gain shows good agreement with the simulation. At the peak point near 140 m, the gain enhancement reaches 12.1 dB resulting in 15.9 dB overall parametric gain. The drop in gain near the peak point can be ascribed to the increased polarization effect from extended fiber length. In both 2- and three-stage cases, the contribution of gain from a SMF-28 fiber section is less than 1 dB. Note that the gain in second or third stages (~6 dB) is almost twice the maximum achievable gain of the first stage (~3.5 dB) in log scale. This increase in maximum achievable gain results from the involvement of conjugate wave, which is absent at the input of the first stage, in the PA process.

IV. DISCUSSION

We numerically investigate the influence of QPM on PA gain spectrum for future applications in multi-wavelength systems. The simulation parameters are chosen in accordance with the experiment (QPM target wavelength at 5 nm away from λ_p). Perfect QPM is enforced by resetting the relative phase difference of the target wavelength component to zero at each L_{max} . We simulate up to the fifth stage and present the results in Fig. 7. At 5 nm wavelength offset, we obtain 28 dB gain enhancement with five stages. The gain spectra retain the original profile with only 3.2 nm reduction in 3 dB gain bandwidth.

Comparisons with previously reported experimental results are also performed to estimate the usefulness of the scheme. The 15.9 dB overall gain obtained with 17 W pump power, 150 m of DS fiber, and 11 nm $\lambda_o - \lambda_p$ separation is comparable to the 18 dB maximum gain in [2] achieved with 7 W pump power, 200 m of DS fiber, and 0.8 nm $\lambda_o - \lambda_p$ separation. Our result is also comparable to 20 dB maximum gain in [4] obtained with 8 W pump power, 40 m of high nonlinearity fiber, and 49 nm $\lambda_o - \lambda_p$ separation when an order of magnitude difference in fiber γ is taken into consideration. The comparison shows that it is possible to enhance PA gain with large $\lambda_o - \lambda_p$ separation by utilizing QPM technique.



Fig. 6. Experimental and simulation results of three-stage QPM PA.



Fig. 7. Gain spectra of QPM PA with various numbers of stage.

The validity and limitation in theory are also examined based on experimental results. Figs. 5 and 6 show good agreement between theory and experiment at output points and along the fiber, respectively. Main deviation occurs near the end of the third stage (see Fig. 6). The limiting factors include: the loss from splicing two different types of fibers, the polarization effect, and technical difficulty of setting the initial phase relations between the pump and signal waves. Experimental investigation for the plausibility of 4~5 stage QPM PA is a natural extension of this work. The validity of the theory implies that the phase relations in parametric processes can be accurately tracked and manipulated. This QPM technique may be adopted in modulation instability regime (where $\lambda_p > \lambda_o$) to cope with the deviation from self-phase matched regime cause by pump depletion.

V. CONCLUSION

We have studied the theory and implementation of QPM PA. The theory of QPM in FWM developed in this work reveals that the QPM can be achieved by ensuring the proper initial phase relation and then by introducing additional relative phase difference $\pm \pi$ at every $L_{\text{max}} = \pi/2|g|$. We experimentally show the validity of the theory with two- and three-stage QPM PA. The two-stage QPM PA experiments with varied amount of relative phase difference verify the theory. The three-stage QPM PA experimental results also corroborate the theory by showing good agreement with the simulation. We achieve 15.9 dB overall

gain and 12.1 dB gain enhancement over non-QPM PA with the three-stage QPM PA. The simulation of gain spectrum shows that the QPM induces only 3.2 nm reduction in 3 dB gain bandwidth even with >20 dB enhancement in parametric gain.

We suggest that QPM is a simple and effective method for enhancing the gain of fiber-optic PA while maintaining a wide separation between pump and zero-dispersion wavelength. Since the QPM results in flat gain spectrum between QPM target wavelengths, it can find the best application for wide band optical amplification in multi-wavelength communication systems.

REFERENCES

- R. H. Stolen and J. E. Bjorkholm, "Parametric amplification and frequency conversion in optical fibers," *IEEE J. Quantum Electron.*, vol. QE-18, pp. 1062–1072, July 1982.
- [2] M. E. Mahric, N. Kagi, T.-K. Chiang, and L. G. Kazovsky, "Broadband fiber optical parametric amplifiers," *Opt. Lett.*, vol. 21, pp. 573–575, Apr. 1996.
- [3] G. A. Nowak, Y.-H. Kao, T. J. Xia, and M. N. Islam, "Low-power highefficiency wavelength conversion based on modulational instability in high nonlinearity fiber," *Opt. Lett.*, vol. 23, pp. 936–938, Jun. 1998.
- [4] M. E. Mahric, F. S. Yang, M.-C. Ho, and L. G. Kazovsky, "High-nonlinearity fiber optical parametric amplifier with periodic dispersion compensation," *J. Lightwave Technol.*, vol. 17, pp. 210–215, Feb. 1999.
- [5] M. Karlsson, "Four-wave mixing with randomly varying zero-dispersion wavelength," J. Opt. Soc. Amer. B, Opt. Phys., vol. 15, pp. 2269–2275, Aug. 1998.
- [6] K. Inoue, "Arrangement of fiber pieces for a wide wavelength conversion range by fiber four-wave mixing," *Opt. Lett.*, vol. 19, pp. 1189–1191, Aug. 1994.
- [7] H. Takahashi and K. Inoue, "Cancellation of four-wave mixing by use of phase shift in a dispersive fiber inserted into a zero-dispersion transmission line," *Opt. Lett.*, vol. 20, pp. 860–862, Apr. 1995.
- [8] F. Matera, A. Mecozzi, M. Romagnoli, and M. Settembre, "Sideband instability induced by periodic power variation in long-distance fiber links," *Opt. Lett.*, vol. 18, pp. 1499–1501, Sept. 1993.
- [9] K. Kikuchi, C. Lorattanasane, F. Futami, and S. Kaneko, "Observation of quasi-phase matched four-wave mixing assisted by periodic power variation in a long-distance optical amplifier chain," *IEEE Photon. Technol. Lett.*, vol. 7, pp. 1378–1381, Nov. 1995.
- [10] I. Bar-Joshep, A. A. Friesem, R. G. Waarts, and H. H. Yaffe, "Parametric interaction of a modulated wave in a single-mode fiber," *Opt. Lett.*, vol. 11, pp. 534–536, Aug. 1986.
- [11] A. Vatarescu, "Light conversion in nonlinear monomode optical fibers," J. Lightwave Technol., vol. LT-5, pp. 1652–1659, Dec. 1987.

Jaeyoun Kim received the B.S. degree from Kwangwoon University, Seoul, Korea, in 1992 and the M.S. degree from the University of Arizona, Tucson, in 1994, both in electrical engineering. Since 1997, he has been pursuing the Ph.D. degree in electrical engineering at the University of Michigan at Ann Arbor as an EECS Department Fellow.

From 1995 to 1997, he was with the Institute for Advanced Engineering, Seoul, as a Research Engineer. At the Institute, he was involved in research and development of laser-/fiber-optic gyroscopes. Currently, he is a Research Assistant in the Integrated Optics Laboratory, University of Michigan. His research interests include linear and nonlinear aspects of fiber optics, design, simulation, and fabrication of integrated optical devices, and utilization of them in optical transmission systems.



Özdal Boyraz (S'96) received the B.S. degree in electrical engineering from the Hacettepe University, Turkey, in 1993, and the M.S. degree in electrical engineering degree from the University of Michigan, Ann Arbor, in 1997. He is currently pursuing the Ph.D. degree in electrical engineering at the University of Michigan, Ann Arbor. His research areas include linear and nonlinear aspects of fiber optic communication systems. His Ph.D. study includes experimental study of all-optical switching and access node design, optical WDM sources, and

optical analog-to-digital conversion.

Mr. Boyraz was awarded a full scholarship by the Turkish government for his graduate studies.

Jin H. Lim received the Ph.D. degree in optics from CREOL, University of Central Florida, Orlando, in 1998. In 1999, he became a Postdoctoral Research Fellow at EECS in the University of Michigan, Ann Arbor.

In June 2000, he joined Lucent Technologies, Microelectronics, as a Member of Technical Staff in the Optoelectronics Center. He is currently conducting the research and development on optical fiber amplifiers for DWDM system.

Mohammed N. Islam received the B.S. degree in 1981, the M.S. degree in 1983, and the Sc.D. degree in 1985, all in electrical engineering, from the Massachusetts Institute of Technology, Cambridge.

From 1985 to 1992, he was a Member of the Technical Staff in the Advanced Photonics Department at AT&T Bell Laboratories, Holmdel, NJ. In 1992, he joined the EECS Department, the University of Michigan, Ann Arbor, where he is currently a Full Tenured Professor. He has published over 95 papers in refereed journals and holds over 25 patents.

Prof. Islam was a Fannie and John Hertz Fellow from 1981 to 1985, and in 1992 was awarded the OSA Adolf Lomb Medal for pioneering contributions to nonlinear optical phenomena and all-optical switching in optical fibers. He also received the U-M Research Excellence Award in 1997 and became a Fellow of the Optical Society of America in 1998. In addition, he has founded two spin-off companies from the University of Michigan: AccuPhotonics and Bandwidth Solutions.